B.Sc. (CBCS Pattern) Semester-VI

021A - (DSE-VI) - Mathematics Paper-II : Complex Analysis and Vector Calculus

P. Pages : 3 **GUG/S/25/13361**

Time : Three Hours Max. Marks : 60

* 1 7 2 6 *

Notes : 1. Solve all the **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) If $f(z)=u(r,\theta)+iv(r,\theta)$ is an analytic functions in Z-plane, then prove that **6**

$$u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}=0$$

b) If $f(z)=u(x,y)+iv(x,y)$ is analytic function of Z, then prove that $u=c_1$ and $v=c_2$ intersect orthogonally. Hence find the equation of curve orthogonal to $y^3-3x^2y=c_1$. **6**

OR

c) Prove that energy bilinear transformation with two non-infinite fixed points α,β is of the form $\frac{w-\alpha}{w-\beta}=k\frac{z-\alpha}{z-\beta}$ where K is a constant. **6**

d) Find the fixed points of the bilinear transformation **6**
$w=\frac{(2+i)z-2}{i+z}$ what is its normal form? show that the transformation is Loxodromic.

UNIT – II

2. a) Evaluate $\int_{1+i}^{2+4i} z^2dz$ along $x=t,\ y=t^2$ where $1\le t<2$. **6**

b) If f(z) is analytic in a simply connected region D, then rove that $\int_c f(z)dz$ is independent of path in D. **6**

OR

c) Let f(z) be analytic in a simply connected domain D and C be a simple closed curve in D oriented counter clockwise. then prove that for any point a within C. **6**

$$\oint_C \frac{f(z)}{z-a}dz=2\pi i f(a).$$

d) Evaluate $\int_C \frac{z^2}{(z-1)^2(z+2)}dz$, when C is the circle |z| = 3. by using Cauchy residue theorem. **6**

UNIT – III

3. a) If $\phi = x^3 + y^3 + z^3 - 3xyz$. Find div grad ϕ and curl grad ϕ. **6**

b) Find the unit tangent vector at any point on the curve $x = t^2 + 2,\ y = 4t - 5,\ z = t^2 - 2t$. Determine the unit tangent at the point t = 2. **6**

OR

c) If $\overline{F} = \left(2x + y^2\right)\overline{i} + \left(3y - 4x\right)\overline{j}$, evaluate $\int_C \overline{F} \cdot d\overline{r}$ around the path $y = x^2$ joining (0,0) and (1,1) **6**

d) Compute the line integral $\int_C y^2 dx - x^2 dy$, about the triangle whose vertices are (1,0), (0,1) and (-1,0). **6**

UNIT – IV

4. a) Let R be a closed bounded region in the xy-plane whose boundary is a simple closed curve C which may be cut by any line parallel to the co-ordinate axes in at most two points. Let M(x,y) and N(x,y) be functions that are continuous and have continuous partial derivatives $\frac{\partial M}{\partial y}$ and $\frac{\partial N}{\partial x}$ in R. Then prove that **6**

$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\, dy = \oint_C \left(M dx + N dy\right).$$

where C is traversed in the positive direction.

b) Show that $\iint_S \left(ax\overline{i} + by\overline{j} + cz\overline{k}\right) \cdot \hat{n}\, dx = \frac{4}{3}\pi\left(a + b + c\right)$ where S is the surface of sphere $x^2 + y^2 + z^2 = 1$ by using divergence theorem. **6**

OR

c) Evaluate $\iint_S \overline{f} \cdot \hat{n}\ dS$, where $\overline{f} = x^2\overline{i} + y^2\overline{j} + z^2\overline{k}$ and S is the surface of the solid cut off by the plane x + y + z = a from the first octant. **6**

d) Verify stokes theorem for the vector field defined by $\overline{f} = \left(x^2 - y^2\right)\overline{i} + 2xy\overline{j}$ in the rectangular region in the xy plane bounded by lines x = 0, x = a, y = 0, y = b. **6**

5. Solve **any six.**

a) Prove that $u = e^{-x}\left(x \sin y - y \cos y\right)$ is harmonic. **2**

b) Find fixed points of bilinear transformations $w = \frac{az + b}{cz + d},\ ad - bc \neq 0$. **2**

c) Show that $\int_C \frac{dz}{Z-a} = 2\pi i$, where – C' is the circle with center a and radius r. **2**

d) Evaluate $\oint_C \frac{dz}{\cosh z}$, where C is the circle | z | = 2 by using Cauchy integral theorem. **2**

e) Find $\nabla\phi$, if $\phi = 2x^2y^3 - 3y^2z^3$ at the point (1,-1,1). **2**

f) Define surface integral as a limit of sum. **2**

g) Show that $\iint_S \bar{r}.\hat{n}\, dS = 3V$, where V is the volume enclosed by S. **2**

h) State stokes theorem. **2**

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